$$H:R::R:R-h \text{ or } H=R\sqrt{2}.$$

Now by the law of gravitation, $(R\sqrt{2})^2:R^2::150:x$, the required weight, that is, $x=\dfrac{150R^2}{2R^2}=75$.

112. Proposed by **F. P. MATZ. M. Sc., Ph. D.,** Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pa.

Vary the radius of curvature of a plane curve inversely as the abscissa; then the solution will give you, (1) Ryan's Equation of the Elastic Curve, and (2) Wood's Equation of the Hydrostatic Curve.

Solution by **G. B. M. ZERR, A. M., Ph. D.,** Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$$\text{Radius of curvature}=\frac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}=\frac{a^2}{2x}.$$

$\therefore\ a^2(d^2y/dx^2)=2x[1+(dy/dx)^2]^{\frac{3}{2}}$. Let $dy/dx=p$.

$\therefore\ a^2(dp/dx)=2x(1+p^2)^{\frac{3}{2}}$ or $a^2p/\sqrt{(1+p^2)}=(x^2+A)$.

$\therefore\ p=dy/dx=\pm\sqrt{(x^2+A)}/\sqrt{[a^4-(x^2+A)^2]}$. Let $x^2+A=a^2\cos\theta$.

$$\therefore\ y=\mp\tfrac{1}{2}a^2\int\frac{\cos\theta d\theta}{\sqrt{(a^2\cos\theta-A)}}=\mp\tfrac{1}{2}a^2\int\frac{(1-2\sin^2\frac{1}{2}\theta)d\theta}{\sqrt{(a^2+1-A-2\sin^2\frac{1}{2}\theta)}}.$$

Let $\frac{1}{2}\theta=\varphi$ and $2/(a^2+1-A)=e^2$.

$$\therefore\ y=\pm\frac{a^2e^2}{2}\int\frac{(1-2\sin^2\varphi)d\varphi}{\sqrt{(1-e^2\sin^2\varphi)}}.$$

$\therefore\ y=\pm a^2E(e,\ \varphi)\mp(\frac{1}{2}a^2)(2-e^2)F(e,\ \varphi)+B\ldots.(1)$.

Since $\varphi=\frac{1}{2}\theta$, $y=\pm a^2E(e,\ \frac{1}{2}\theta)\mp(\frac{1}{2}a^2)(2-e^2)F(e,\ \frac{1}{2}\theta)+B\ldots.(2)$.

(1) represents Ryan's Elastic Curve and (2) represents the Hydrostatic Curve. In the above $e<1$ and $A<a^2+1$.

In the second equation the curve can never cross the line of force since $\sin(\frac{1}{2}\theta)$ cannot equal $1/e$.

Also solved by *J. SCHEFFER*.

DIOPHANTINE ANALYSIS.

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58. Proposed by **JOHN M. COLAW, A. M.,** Monterey, Va.

Find a square proper fraction which if subtracted from unity will leave for remainder a square proper fraction.

I. Solution by C. E. ARMENTROUT, Professor of Mathematics, Rockingham Military Institute, Mt. Crawford, Va.; G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; and P. S. BERG, Larimore, N. D.

Let $(y/x)^2$ be the square proper fraction.

$1-(y/x)^2=(n/m)^2$ or $m^2(x^2-y^2)=n^2x^2$.

Let $x=p^2+q^2$, $y=p^2-q^2$.

$\therefore\ 1-\left(\dfrac{p^2-q^2}{p^2+q^2}\right)=\dfrac{4p^2q^2}{(p^2+q^2)^2}$, where p can be any value greater than q.

Let $p=2$, $q=1$. $\therefore\ 1-(\frac{3}{5})^2=(\frac{4}{5})^2$.

Let $p=3$, $q=2$. $\therefore\ 1-(\frac{5}{13})^2=(\frac{12}{13})^2$.

II. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C., and COOPER D. SCHMITT, A. M., University of Tennessee, Knoxville, Tenn.

This reduces to the simple statement that $a^2/c^2+b^2/c^2=1$, or that $a^2+b^2=c^2$. This is evidently satisfied in the case of every rational right triangle.

Thus, $3^2+4^2=5^2$, or $\frac{9}{25}+\frac{16}{25}=1$.

This is the same as saying $1-\frac{9}{25}=\frac{16}{25}$, thus fulfilling the condition of the problem.

Similarly an indefinite number of examples might be given.

Also solved by *J. H. DRUMMOND*, *ALOIS F. KOVARIK*, *J. W. YOUNG*, and the late *SYLVESTER ROBINS*.

86. Proposed by A. H. BELL, Hillsboro, Ill.

The edges of a rectangular parallelopiped are within one of the proportion 3:6:7, and if they are $3x$, $6x\mp1$, $7x$, then $(3x)^2+(6x\mp1)^2+(7x)^2=$the diagonal squared$=94x^2\mp12x+1=\square$. To find four integral values for x in this equation.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $94x^2\mp12x+1=(mx\mp1)^2=m^2x^2\mp2mx+1$.

$\therefore\ x=\pm\dfrac{2(m-6)}{94-m^2}$.

Expanding $\sqrt{94}$ as a continued fraction we get

$$\sqrt{94}=9+\frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{3+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{5+}\,\frac{1}{1+}\,\frac{1}{8+}\,\frac{1}{1+}\,\frac{1}{5+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{3+}\,\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{9+}$$

The successive convergents are as follows :

$$\frac{9}{1},\quad \frac{10}{1},\quad \frac{29}{3},\quad \frac{97}{10},\quad \frac{126}{13},\quad \frac{223}{23},\quad \frac{1241}{128},\quad \frac{1644}{151},\quad \frac{12953}{1336},\quad \frac{14417}{1487},$$

$$\frac{85038}{8771},\quad \frac{99455}{10258},\quad \frac{184493}{19029},\quad \frac{652934}{67345},\quad \frac{1490361}{1537191},\quad \frac{2143295}{221064},\ \ldots\ldots$$